Audio Engineering Society

Convention Paper

Presented at the 141st Convention
2016 September 29 – October 2, Los Angeles, CA, USA

Force Factor Modulation in Electro Dynamic Loudspeakers

Lars Risbo[1], Finn T. Agerkvist[2], Carsten Tinggaard[3], Morten Halvorsen[3], and Bruno Putzeys[1]

[1]*Purifi, Denmark/Belgium*
[2]*Acoustic Technology, DTU-Elektro, Technical University of Denmark, DK-2800 Lyngby, Denmark*
[3]*PointSource Acoustics, Roskilde, Denmark*

Correspondence should be addressed to Lars Risbo (lars@purifi.dk)

ABSTRACT

The relationship between the non-linear phenomenon of 'reluctance force' and the position dependency of the voice coil inductance was established in 1949 by Cunningham, who called it 'magnetic attraction force'. This paper revisits Cunningham's analysis and expands it into a generalised form that includes the frequency dependency and applies to coils with non-inductive (lossy) blocked impedance. The paper also demonstrates that Cunningham's force can be explained physically as a modulation of the force factor which again is directly linked to modulation of the flux of the coil. A verification based on both experiments and simulations is presented along discussions of the impact of force factor modulation for various motor topologies. Finally, it is shown that the popular L_2R_2 coil impedance model does not correctly predict the force unless the new analysis is applied.

1 Introduction

The electro-dynamic loudspeaker recently celebrated its centenary and has not materially changed construction since the direct radiating speaker by Rice and Kellogg in 1923. Production of loudspeakers today numbers in billions of units per year and thanks to digital audio storage and distribution the speaker is by orders of magnitude the most non-linear device in the audio chain. Unfortunately, the in-depth understanding and modelling of speakers has progressed at a modest pace and leaves still much work to be done.

A major breakthrough was the work by Cunningham in 1949 [1] who analysed the inherently non-linear response of the motor due to magnetic effects with surprising depth of insight. Aside from the position dependent force factor due to the non-homogeneous field in the gap and the linearising effect of overhung coils, Cunningham analysed 'Distortion due to magnetic attraction forces'. He showed that 'this effect is not dependent upon the presence of a permanent field' but instead that this force was given by [1] :

$$F_i = \frac{\partial W_{m,i}}{\partial x} = \frac{\partial}{\partial x}\left(\tfrac{1}{2}i^2 \cdot L_e(x)\right) = \tfrac{1}{2}i^2 \cdot L_e'(x) = i \cdot Bl_i, \tag{1}$$

where $W_{m,i}$ is the stored magnetic energy, i is the coil current, $L_e(x)$ is the position dependent inductance of the voice coil and Bl_i is defined as the current dependent force factor. The force is proportional to the square of the current and the spatial gradient of the coil's inductance. The result is 2nd order distortion as well as a DC force acting in the direction of highest inductance.

Cunningham considered only moving coil transducers, but we will show in the Appendix that the analysis is equally applicable to all 3 known motor type, namely 'moving coil', moving magnet' and 'moving iron'. All three motor types produce a force in response to the coil current having a desired proportional component defined by a permanent force factor as well as an undesired quadratic response c.f. (1). We will follow the conventions of moving coil transducers and call the force factor Bl. This does not affect the generality of our analysis.

The flux in the coil is central to the analysis of the force as we will show and the term flux modulation due to the coil current is thus quite apt. Still we prefer the term force factor modulation since the force factor is the most important characteristic of a black box model of a motor while the magnetic field making up the flux is a complex 3-dimensional internal characteristic. Moreover, the term 'flux modulation' is sometimes used to refer to a change in permeability due to saturation of the iron [2]. The brevity of Cunningham's analysis (basically just one short paragraph) may be the reason for today's apparent confusion where 'flux-modulation' and 'reluctance force' are often treated as separate phenomena (see e.g., [3]).

The drive towards speakers with long strokes, full audio range and high linearity in very small form factors makes Cunningham's analysis even more important today than in 1949 when low power amplifiers dictated the use of speakers with large diaphragms and low excursion. Force factor modulation is a much greater problem today.

1.1 Paper Structure

A fundamental analysis of the physics of a generalised electro-magnetic machine that serves as a basis for this paper is given in the Appendix. Throughout the paper we assume the use of linear magnetic materials. Section 2 generalises Cunningham's work to cover lossy coils and to include the frequency dependent dynamics of the force. Section 3 tests the theory with both measurements and Finite Element Simulations. Section 4 discusses the new results as applied to the popular lumped parameter models for the speaker-impedance.

2 Generalisation of Cunninghams's 1949 Formula

Equation (1) says that the force F_i produced by the force factor modulation is the spatial gradient of the stored magnetic energy. This equation holds generally as shown by the analysis in the Appendix. Equally fundamental is that the stored magnetic energy due the coil current i and its generated flux Φ_i is:

$$W_{m,i} = \frac{1}{2} i \cdot \Phi_i \tag{2}$$

We will now use the generality of (1) and (2) to study the force when the coil is not a pure inductor but exhibits frequency dependent losses, e.g., from eddy currents. The impedance of speaker coils has been studied intensely in literature [4, 5, 3, 6] .

The first step is to combine (1) and (2) to express the force as a product of the current i and the current dependent force factor Bl_i (also found as (25) in the Appendix):

$$F_i = i \frac{1}{2} \frac{\partial \Phi_i}{\partial x} = i \cdot Bl_i \tag{3}$$

From Faraday's law, the current dependent flux Φ_i can be found from the time integral of the induced voltage in the coil at a stationary (blocked) position x:

$$\Phi_i(t) = \int v_i(t) dt, \tag{4}$$

where $v_i(t)$ is the voltage induced in the coil, i.e., the voltage on the coil minus the voltage across its DC-resistance R_e. We now move to the s-domain (Laplace domain) where $s = j2\pi f$ to express the flux $\Phi_i(s)$ and note that the flux and induced voltage are linear responses of the current (thanks to our assumption of linear magnetic media). In the Laplace domain the time integral is replaced by a division by s:

$$\Phi_i(s) = I(s) \frac{(Z_b(s) - R_e)}{s}, \tag{5}$$

where $Z_b(s)$ is the blocked impedance of the coil and $I(s)$ is the Laplace tranform of the current.

It is now practical to define the generalised inductance L_{gen}:

$$L_{gen}(s) \equiv \frac{Z_b(s) - R_e}{s} \tag{6}$$

Combining (3), (5) and (6) reveals that the instantaneous force factor due to the current $Bl_i(t)$ is the coil current filtered by a transfer function:

$$Bl_i(s) = I(s) \cdot \frac{1}{2} \frac{\partial L_{gen}(s)}{\partial x} = I(s) \cdot H_{Bl}(s), \tag{7}$$

where we defined the force factor transfer function $H_{Bl}(s)$. The current dependent instantaneous force factor $Bl_i(t)$ is simply the coil current $i(t)$ filtered by this transfer function.

We now have a generalisation of Cunningham's formula where the gradient of the inductance is generalised to a filter $H_{Bl}(s,x)$ being the x-gradient of the generalised inductance $L_{gen}(s,x)$. The force factor transfer function $H_{Bl}(s,x)$ represents a dynamic linear system with possible frequency dependent phase and magnitude response. It is noted that for a purely inductive coil (Cunningham's original work) we have that $L_{gen}(s,x) = L_e(x)$. In this special case, $H_{Bl}(s)$ is simply a constant.

The current dependent force factor $Bl_i(t)$ is the instantaneous proportionality between the current dependent force component F_i and the current. However, as shown in the Appendix and dictated by conservation of energy, the back EMF generated in the coil in response to motion of the coil is counter-intuitively *twice* the Bl_i factor times the velocity $x'(t)$ c.f. (27).

A further and very practical consequence of the analysis is that the dynamic force factor modulation effect can simply fully be characterised by measuring (or simulating) the position dependency of the blocked impedance $Z_b(s,x)$. It is not necessary to measure (or simulate) the modulation of the exact magnetic field to know the impact on the force and back EMF.

A generalisation of Cunningham was attempted in [7]. However, only the real part of the generalised inductance (corresponding to the imaginary part of the impedance) was taken into consideration. This means that the force factor modulation caused by the position dependency of the resistive part of the coil impedance is ignored.

2.1 Symptoms of Bl-modulation

For a sinusoidal coil current of $i(t) = A \cdot cos(\omega \cdot t)$ the force produced is for the permanent field $F_0(t) = A \cdot Bl_0(x) \cdot cos(\omega t)$ plus a contribution due to the force factor modulation of:

$$F_i(t) = \frac{1}{2}A^2\Re(H_{Bl}(\omega))(1+\cos(2\omega \cdot t)) + \frac{1}{2}A^2\Im(H_{Bl}(\omega))sin(2\omega \cdot t) \quad (8)$$

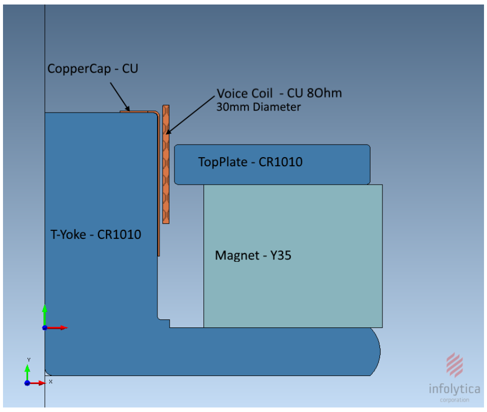

Fig. 1: Cross section of motor structure of the used 4" driver. The motor has rotation symmetry around the x-axis indicated by a vertical line.

The force has both a DC component (caused by real part of H_{Bl}) and a 2^{nd} harmonic component. The imaginary part of $H_{Bl}(s)$ represents the position gradient of the coil losses (effective series resistance) and this also causes force factor modulation in the form of a 2^{nd} harmonic but with no accompanying DC component. Force factor modulation by a low frequency tone will amplitude modulate (AM) a high frequency voice tone (a.k.a. IMD2). AM modulation manifests itself as sidebands to the voice tone (at the sum and difference frequencies) at an amplitude relative to the voice tone of:

$$G_{AM} = \frac{|H_{Bl}(f_{bass}) + H_{Bl}(f_{voice})|A_{cur}}{2 \cdot Bl_0}, \quad (9)$$

,where A_{cur} is the amplitude of the bass tone current and Bl_0 is the linear force factor due to the permanent field.

3 Verification Using Finite Element Simulations and Measurements

The theoretical results in the previous sections were verified using both numerical simulations and measurements. A 4", 8 Ω driver with a motor c.f. Fig. 1 was simulated and built. It is a variant of the transducer used in earlier work [8].

3.1 Finite-Element Simulations

Infolytica's MagNet tool was used for a series of 100 ms transient simulations of a voltage step on the coil for a range of blocked coil positions (−5 mm to

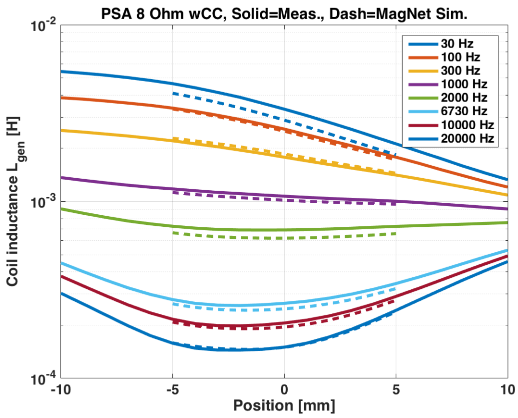

Fig. 2: $|L_{gen}(f,x)|$ plotted versus x with f as parameter. Solid: measurements, Dashed: MagNet simulations.

5 mm in 1 mm increments). The voltage, current and force acting on the voice coil were sampled at 384 kHz. An antialias filter was included in the simulation. The result was exported to Matlab for post-processing. The instantaneous force factor $Bl(t)$ was found as the force divided by the current. Subtracting the initial value $Bl(t=0)$ representing the linear force factor yields the dynamic (current-induced) force factor $Bl_i(t)$. Next the blocked impedance $Z_b(x,s)$ and $H_{Bl}(s,x)$ (i.e., the current to force factor transfer function) were identified using FFT-based deconvolution.

3.2 Comparing Measurements and Simulations of the Blocked Impedance

A precision positioning stage [8] was used to position and hold the coil in the motor structure without membrane or suspension. A periodic noiselike stimulus was applied and current and voltage recorded with a sound card at a 44.1 kHz sampling rate. Impedance vs. frequency curves were estimated using a synchronous FFT. The measurement was repeated for a range of positions (from −10 mm to 10 mm in 1 mm increments). The generalised inductance $L_{gen}(f,x)$ c.f. to (6) was calculated for both the measurements and MagNet simulation results and plotted for comparison in Fig. (2). A convincing match is seen in general. At the lowest frequency (30 Hz) the MagNet result underestimates inductance due to the limited length of the transient simulation. A mechanical resonance around 4 kHz affects the measured result at 2 kHz.

3.3 Numerical Verification of H_{Bl} Using the MagNet Simulations

A 6$^{\text{th}}$ order polynomial fit was used to interpolate the simulated blocked impedance between the discrete x positions. From the interpolated $Z_b(f,x)$, the generalised inductance was found using (6). This was then used to calculate the complex $H_{Bl}(s,x)$ from (7). Fig 3 shows a convincing match between the force factor transfer function $H_{Bl}(s,x)$ and the one obtained from the simulated actual force, both in magnitude and phase and across all positions. This strongly supports the theoretical result of (7). Some discrepancy is observed at higher frequencies around $x=-2$ mm. This is because the 1 mm resolution of the simulated data set is insufficient to reconstruct the sharp minimum of the inductance caused by the copper cap. At this minimum the gradient of the inductance v.s. position changes polarity. The H_{Bl} peaks near the rest position towards DC and droops at both positive and negative position. Some asymmetry is noted: force factor modulation is greater at negative positions (coil inside the motor) than positive positions. This agrees with earlier findings [8].

The rest position $|H_{Bl}|$ peaks at $0.13\,\text{N/A}^2$ at 30 Hz. At this frequency a drive current of 10 A peak (e.g. when the driver is driven hard in a small box) gives an Bl_i amplitude of 1.3 N/A which is about 16 % of the permanent Bl_0 of 8 N/A. Such error is comparable to or even greater than the typical position dependent variation of the permanent $Bl_0(x)$

3.4 Measurement of the DC Force

Force factor modulation produces a DC force in proportion to the real value of H_{Bl} when the coil is driven by a sinusoidal current c.f. (8), i.e., AC current causes a DC force. One would readily assume this DC force always to point inwards [2] since the inductance grows when the coil is pushed into the motor. A surprise prediction of our analysis is that at high frequencies a shorting device (e.g. a copper cap or shorting ring) can modify the inductance gradient to such an extent that the sign changes. When that happens the DC force propels the cone *outwards*. Fig. 4 shows the real and imaginary parts of $H_{Bl}(f)$ at the rest position ($x=0$) for the test driver obtained from the MagNet simulations. The sign of the real part of $H_{Bl}(f)$ changes around 500 Hz.

An experiment was done to measure the DC force caused by an AC current on the 4" test driver (DUT)

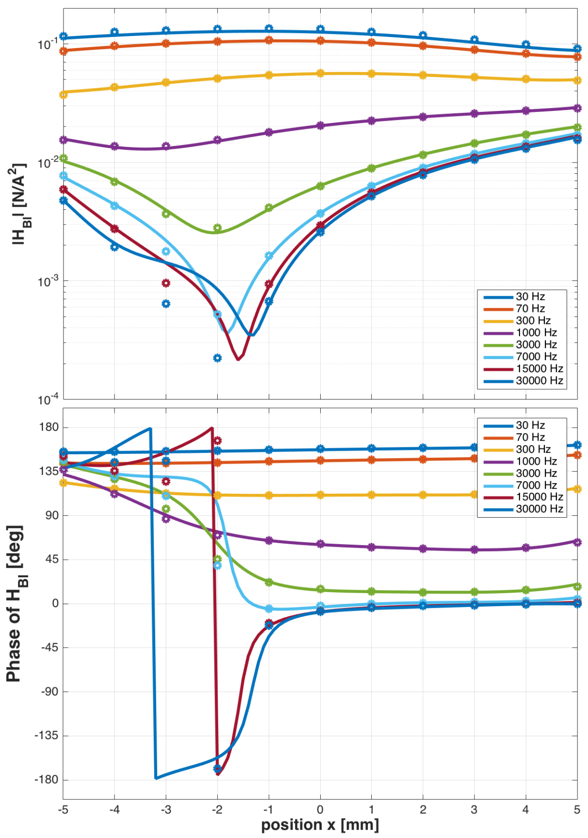

Fig. 3: The force factor modulation transfer function $H_{Bl}(f,x)$, Solid: from the simulated Z_b, Round markers: from the simulated force.

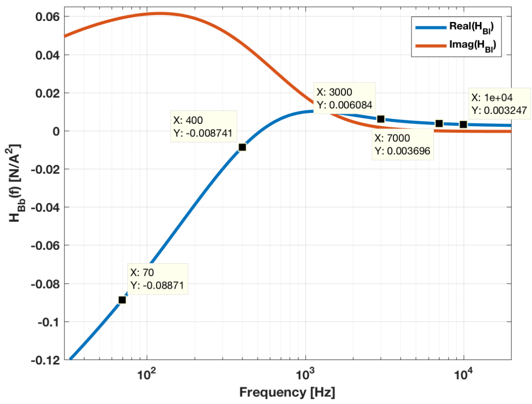

Fig. 4: Real and Imaginary components of $H_{Bl}(f)$for $x = 0$ from MagNet simulations of the 4" test driver.

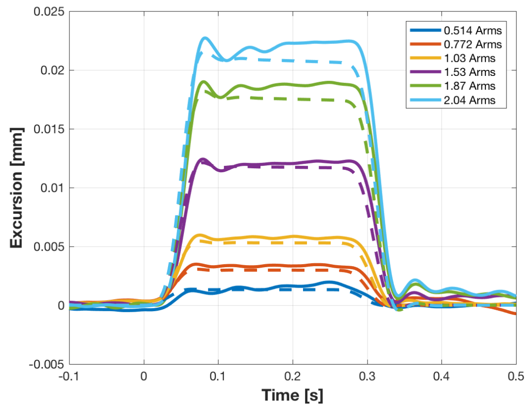

Fig. 5: DC-excursion caused by a series of 7 kHz tone bursts of varying amplitude with a 4th order 20Hz low-pass filter applied. Solid=measured, Dash=model fit.

while minimizing AC excursion. DC terms generated by asymmetries in the permanent force factor $Bl_0(x)$ and suspension compliance $C_{ms}(x)$ [2] might otherwise confound the test.

For the high frequency test the moving mass naturally renders excursion negligible so the driver could simply be tested in free air. For the tests at low frequencies, AC cone movement was countered by another 4" driver mounted in the same cabinet and driven with a signal of the same frequency but with a carefully adjusted phase and amplitude. The cabinet was intentionally made slightly leaky so that again, long term DC excursion was controlled only by the free-air complince of the driver. For all experiments, the AC excursion remained below 10 μm in amplitude. A 470 μF DC blocking capacitor was added in series with the voice coil.

Excursion was measured with a Keyence triangulating laser head and captured alongside coil current. DC free air compliance was estimated at C_{ms} =1.25 mm/N by applying a 10 gram weight and recording displacement, permitting conversion between DC excursion and DC force.

Fig. 5 shows the result for a series of 7 kHz tone bursts at different amplitudes. The recorded excursion was low-pass filtered at 20 Hz. For comparison the graph is overlaid with a plot of the predicted force multiplied by the estimated compliance. To be precise, the predicted force is the square of the current filtered by the same low-pass filter and scaled by a best fit constant K which ideally equals $\Re(H_{Bl})$. The experiment was repeated for 3 kHz and 10 kHz resulting in:

frequency	best fit K	simulated $\Re(H_{Bl})$
3 kHz	$7.6\times10^{-3}\,\mathrm{N/A^2}$	$6.1\times10^{-3}\,\mathrm{N/A^2}$
7 kHz	$4\times10^{-3}\,\mathrm{N/A^2}$	$3.7\times10^{-3}\,\mathrm{N/A^2}$
10 kHz	$3.6\times10^{-3}\,\mathrm{N/A^2}$	$3.2\times10^{-3}\,\mathrm{N/A^2}$

The measured DC excursion is about 10 % to 15 % larger than expected. Possible contributing error factors is that H_{Bl} is strongly dependent on the exact rest position and that the compliance varies over time.

Next a series of 70 Hz bursts at varying amplitudes were applied. As expected this resulted in a negative DC excursion as shown in Fig. 6. Again the best fit model is overlaid with $K = -0.1\,\mathrm{N/A^2}$. This is quite close to the simulated value $\Re(H_{Bl}(70\,\mathrm{Hz})) = -0.089\,\mathrm{N/A^2}$, i.e., about the same relative error as for the high frequency bursts. However, the excursion is progressively larger than the quadratic model for large currents. Most likely we are seeing our starting assumption, that the magnetic materials are substantially linear, become progressively more inaccurate as the drive current goes up. The slow settling time is dictated by the air leak. Finally, a 400 Hz burst was applied (with active AC excursion cancellation). This frequency was chosen since the real value of H_{Bl} here is very close to zero. Indeed, the plot shows that the excursion settles essentially back to zero after a positive transient.

4 Lumped Parameter Models

In literature several simplified models have been suggested to approximate the way the blocked electrical impedance of a real loudspeaker depends on frequency. Some of them, [4, 5, 6], provide an equation that directly describes the impedance of the lossy inducance

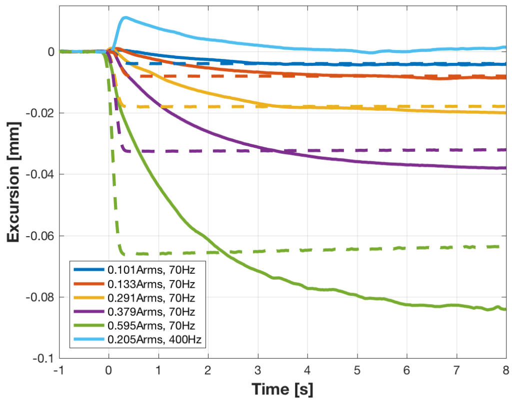

Fig. 6: DC-excursion caused by a series of 70 Hz tone bursts of varying amplitude and a single 400 Hz burst. All plots filtered by a 4th order 20Hz low-pass filter. Solid=measured, Dash=model fit.

and the generalized calculation method presented in this paper can be applied in a straightforward manner on these models, while other authors [11], [12] provide a lumped parameter network.

For the L_2R_2 model, illustrated in Fig. 7, the current in L_2, can be shown to be a low pass filtered version of the current in L_e:

$$I_2(s) = I(s)G(s),\, G(s) = \frac{R_2/L_2}{s + R_2/L_2} \tag{10}$$

which in the time domain becomes

$$i_2(t) = g(t) * i(t), \tag{11}$$

with $\omega_2 = \frac{R_2}{L_2}$ and $*$ represents convolution and $g(t)$ is the impulse response of the filter $G(s)$:

$$g(t) = \begin{cases} \omega_2 e^{-\omega_2 t} & ,t > 0 \\ 0 & ,t \le 0 \end{cases} \tag{12}$$

It is tempting to apply Cunninghams equation directly [11] on each of the two ideal inductances and their respective currents, i and i_2 , which gives

$$F_{i,L_2R_2}(t) = \frac{1}{2}\frac{\partial L_e}{\partial x} i(t)^2 + \frac{1}{2}\frac{\partial L_2}{\partial x} i_2(t)^2 \tag{13}$$

$$= \frac{1}{2}\frac{\partial L_e}{\partial x} i(t)^2 + \frac{1}{2}\frac{\partial L_2}{\partial x} (g(t) * i(t))^2, \tag{14}$$

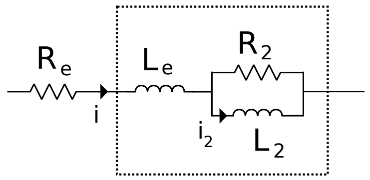

Fig. 7: Model of blocked voice coil impedance using the L_2R_2 model. Dashed box indicate inductive part of impedance.

If it is assumed that R_2 and L_2 covary with displacement the force becomes.

$$F_{i,L2R2} = \frac{1}{2}\frac{\partial L_E}{\partial x} i^2 + \frac{1}{2}\frac{\partial L_2}{\partial x}(g*i)^2 \quad (15)$$

i.e., the current is filtered through $g(t)$ before being squared.

The generalized method of calculation from section 2 is based on the impedance of the network

$$Z_b(s) = R_e + sL_e + \frac{sL_2R_2}{sL_2+R_2} \quad (16)$$

from which the generalised inductance can be calculated as

$$L_{gen}(s) = L_e + L_2G(s) \quad (17)$$

and equations (3) and (7) gives

$$F_i(t) = \frac{1}{2}\frac{\partial L_e}{\partial x} i(t)^2 + \frac{1}{2}\frac{\partial L_2}{\partial x}(g(t)*i(t))\cdot i(t) \quad (18)$$

i.e., the current is filtered once before being multiplied with itself, and force factor transfer function is

$$H_{Bl}(s) = \frac{1}{2}\frac{\partial L_e}{\partial x} + \frac{1}{2}\frac{\partial L_2}{\partial x}G(s) \quad (19)$$

This disagreement on the influence of the filter $g(t)$ is in fact due to an incorrect interpretation of the L_2R_2 model. One should see it as a black box, where the inside details cannot be trusted to give physical meaning. While the model is valid for predicting the blocked impedance, L_2 and in particular its current, i_2, is a model abstraction, which does not exist in the physical system. The physical systems only contains one (lossy) inductance and current, i, and the flux generated by L_e and L_2 are not independent. Consequently one cannot use Cunninghams equation on L_2 directly. However if one goes back to the fundamental equation for the energy stored (3):

$$F_i(t) = i(t)\frac{1}{2}\frac{\partial \Phi(t)}{\partial x} = i\frac{1}{2}\frac{\partial(L_e\cdot i(t) + L_2\cdot i_2(t))}{\partial x} \quad (20)$$

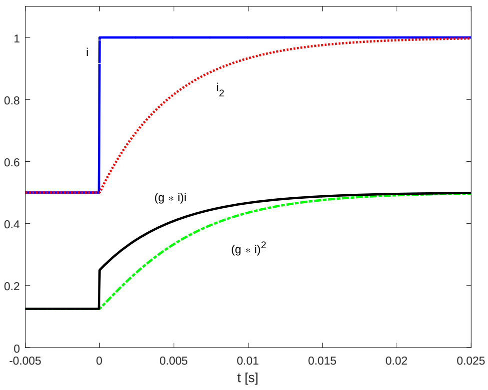

Fig. 8: Currents in the L_2R_2 model and force contributions for L_2 calculated directly on L_2 ($(g*i)^2$) and via the network impedance ($(g*i)i$).

the correct result is obtained in agreement with equation (18), indicating that the total flux ($L_ei + L_2i_2$) is accurately explained by the L_2R_2 model, but the additional energy stored in L_2 is not correct, because the flux generated in L_2 interacts with the flux from L_e.

The difference between the two result is illustrated in Fig. 8. Here the force originating from L_2 only is shown in the case where the current is stepped from 0.5 to 1 amperes. The generalized calculation shows a clear step in the force, whereas the calculation based on equation (14) does not and deviates significantly in the transient response.

The method presented here can also be applied to more advanced lumped parameter models containing more elements such as suggested in [12], so long as the model has a valid connection between the voltage drop over the inductive (excluding R_e) part and the total flux generated.

5 Conclusion

The results of Cunningham can be generalised to comprise all electromagnetic motor/actuator types and include the general frequency dependency of the force factor transfer function for coils with losses. The presented framework highlights the close relationship between the position modulation of the coil's blocked impedance and the force factor modulation. The measurements and numerical simulations support the theory and indicate that the force factor modulation is significant and should not be ignored since it can easily be as large as 10%, i.e., comparable to other large signal errors [2]. The underlying mechanism is that the force factor changes if a change in stored magnetic energy happens when the motor moves position. This storage of energy causes an asymmetry between the force factor and the ratio between velocity and the back-EMF produced by the motor. Finally, it was concluded that the popular L_2R_2 impedance model does not predict the correct dynamic force when the original Cunningham equation is used on the L_2 inductor alone. Continued research must be done in expanding the analysis framework to include non-linear magnetic response.

Acknowledgements

The authors would like to thank Samel Arslanagic from DTU Elektro for fruitful discussions on the electromagnetic analysis and Christian Berg Jacobsen of PointSource Acoustics for preparing the speaker test assembly.

References

[1] Cunningham, W. J., "Nonlinear Distortion in Dynamic Loudspeakers due to Magnetic Effects," *Journal of the Acoustical Society of America*, 21(3), pp. 202–207, 1949.

[2] Klippel, W., "Tutorial: Loudspeaker Nonlinearities—Causes, Parameters, Symptoms," *J. Audio Eng. Soc*, 54(10), pp. 907–939, 2006.

[3] Birt, D. R., "Nonlinearities in Moving-Coil Loudspeakers with Overhung Voice Coils," *J. Audio Eng. Soc*, 39(4), pp. 219–231, 1991.

[4] Vanderkooy, J., "A Model of Loudspeaker Driver Impedance Incorporating Eddy Currents in the Pole Structure," *J. Audio Eng. Soc*, 37(3), pp. 119–128, 1989.

[5] Wright, J. R., "An Empirical Model for Loudspeaker Motor Impedance," *J. Audio Eng. Soc*, 38(10), pp. 749–754, 1990.

[6] Leach, W. M., Jr., "Loudspeaker Voice-Coil Inductance Losses: Circuit Models, Parameter Estimation, and Effect on Frequency Response," *J. Audio Eng. Soc*, 50(6), pp. 442–450, 2002.

[7] Merit, B. and Novak, A., "Magnet-Only Loudspeaker Magnetic Circuits: A Solution for Significantly Lower Current Distortion," *J. Audio Eng. Soc*, 63(6), pp. 463–474, 2015.

[8] Halvorsen, M., Tinggaard, C., and Agerkvist, F. T., "Flux Modulation in the Electrodynamic Loudspeaker," in *Audio Engineering Society Convention 138*, 2015.

[9] Kochendörfer, F. and Voishvillo, A., "Comparative Static and Dynamic FEA Analysis of Single and Dual Voice Coil Midrange Transducers," in *Audio Engineering Society Convention 139*, 2015.

[10] Lastrucci, C., "A Novel Moving Magnet Linear Motor," in *Audio Engineering Society Convention 136*, 2014.

[11] Klippel, W., "Distortion Analyzer-A New Tool for Assessing and Improving Electrodynamic Transducer," in *Audio Engineering Society Convention 108*, 2000.

[12] Thorborg, K. and Futtrup, C., "Electrodynamic Transducer Model Incorporating Semi-Inductance and Means for Shorting AC Magnetization," *J. Audio Eng. Soc*, 59(9), pp. 612–627, 2011.

[13] Voishvillo, A. and Kochendörfer, F., "Application of Static and Dynamic Magnetic Finite Elements Analysis to Design and Optimization of Transducers Moving Coil Motors," in *Audio Engineering Society Convention 135*, 2013.

[14] Cheng, D., *Field and Wave Electromagnetics: Pearson New International Edition*, Pearson Education Limited, 2014, ISBN 9781292038940.

[15] Aldoshina, I. A., Voishvillo, A., and Mazin, V., "Modeling of Flux Modulation Distortion in Moving-Coil Loudspeakers by the Finite-Element Method," in *Audio Engineering Society Convention 98*, 1995.

A Force and Energy Balance of an Electromagnetic Machine

Consider an electromagnetic machine of Fig. 9 having a single coil with flux Φ, no DC resistance and driven by a current source $i(t)$ resulting in the coil voltage

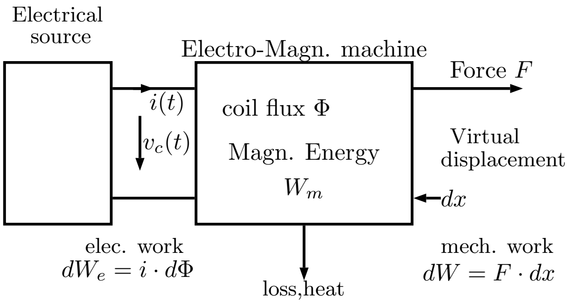

Fig. 9: A generalised single coil electromagnetic machine considered for a virtual displacement dx at constant current $i(t)$. The machine acts with a force F resulting in a work $dW = F \cdot dx$ and consumes electrical energy dW_e whilst storing magnetic energy dW_m.

$v_c(t)$. One or more optional permanent magnets (or energised field coils with constant current) provide a permanent magnetic field in combination with ferromagnetic materials to shape the field. For the purpose of this analysis we assume the magnetic materials to be linear so that the response from current to the magnetic B-field is linear. The machine has a moving part that can move along an x-axis whilst producing a force F acting on its exterior. The moving part of the machine can be the coil itself, a piece of iron or a magnet. Such transducers are called moving coil, moving iron and moving magnet transducers respectively. Without loss of generality a similar analysis could be made for a rotating motor defined by its shaft angle θ and torque T [14].

A common method for finding forces in electromechanical systems is to study the energy balance during a tiny instantaneous 'virtual' displacement dx [14]. The system performs mechanical work equalling $dW = F \cdot dx$. The required energy may come from electrical work W_e supplied by the source or energy W_m already stored in the magnetic circuit. Since the displacement is instantaneous the current is constant and no energy is lost as heat in the meantime. Energy conservation requires that [14] $F \cdot dx = dW_e - dW_m$. Deriving with respect to x yields:

$$F = \frac{\partial W_e}{\partial x} - \frac{\partial W_m}{\partial x} \tag{21}$$

The coil flux Φ is the sum of a current-independent flux component Φ_0 generated by the permanent magnet and a flux component Φ_i which responds linearly to the coil current $i(t)$, i.e., we have $\Phi = \Phi_0 + \Phi_i$. Both components are generally dependent on the position x. The corresponding components of force (F_0 and F_i) and induced voltage ($v_{c,0}$ and $v_{c,i}$) will be treated separately while noting that the principle of linear superposition applies. Because of this, the analysis holds equally for the three major types of electromagnetic transducers (moving coil, moving iron and moving magnet). Despite being vastly different in their physical construction, these transducer types only differ in behaviour in terms of the relative contributions of the current independent and current dependent components.

A.1 The Force and Voltage From the Permanent Field

First, consider the components of force and voltage that arise strictly from the permanent field. Any change in flux Φ_0 (and hence any induced voltage $v_{c,0}$) can only occur as the result of movement. From Faraday's law of induction it follows that the electrical work dW_e is the integral of the product of the current and the induced voltage in the coil: $dW_e = i \int v_c(t)dt = i \cdot d\Phi_0$. Combined with (21) we can express the permanent component of the force factor $Bl_0(x)$ as the position gradient of the coil flux from the permanent field [15].

$$F_0 = i \cdot \Phi_0'(x) - \frac{\partial W_{m,0}}{\partial x} = i \cdot Bl_0(x) + F_x(x), \tag{22}$$

$Bl_0(x)$ is the "classical" force factor which, multiplied with current, produces the force that in an ideal voice coil transducer would be the only one acting on the membrane. For all three motor types, Bl_0 is proportional to the strength of the permanent magnet. Moving iron motors can achieve a very high force factor but only over a short usable x-range by having a short air gap. This principle was used in the first telephone receivers and is still used in hearing aids today.

$F_x(x)$ expresses a force that is strictly position dependent (i.e., elastic) and is due to the position gradient of the stored energy in (21). Put simply it is the attraction force between magnet and iron if one of them is the moving part. In moving coil transducers this term is therefore zero.

Faraday's law of induction gives us the induced voltage on the coil due to movement, i.e., the so-called back Electro Motive Force (EMF):

$$v_{c,0}(t) = \frac{d\Phi_0}{dt} = \Phi_0'(x) \cdot x'(t) = Bl_0(x) \cdot x'(t) \tag{23}$$

Note the positive polarity is due to the definition of the coil voltage being seen from the electrical source in Fig. (9).

A.2 The Force and Voltage from the Current Dependent Field

Magnetic energy $W_{m,i}$ is stored as a result of its current i and the resulting flux Φ_i that the coil creates. Conceptually the stored energy can be found by ramping the current to zero and integrating the electrical work produced, i.e., we take the stored magnetic energy out as electrical work. This gives the very fundamental and general result [14]:

$$W_{m,i} = \frac{1}{2} i \cdot \Phi_i \tag{24}$$

The displacement dx under this constant current condition may cause a change in the flux $d\Phi_i$ which changes the stored magnetic energy by $dW_{m,i} = \frac{1}{2} i \cdot d\Phi_i$. However, the change in flux also results in electrical work delivered by the source equal to $dW_e = i \cdot d\Phi_i$. This means that $dW_{m,i} = 2dW_e$, i.e., only half of the electrical work is stored as magnetic energy and the other half must equal the mechanical work to satisfy the energy balance expressed in (21):

$$F_i = \frac{\partial W_{m,i}}{\partial x} = i \cdot \frac{1}{2} \frac{\partial \Phi_i}{\partial x} = \; i \cdot Bl_i, \tag{25}$$

which defines the current dependent force factor Bl_i responding linearly to the current. The resulting force (by multiplication with the current) becomes a quadratic function of the current.

The induced voltage on the coil (Faraday's law):

$$v_{c,i}(t) = \frac{d\Phi_i}{dt} = \underbrace{\frac{\partial \Phi_i}{\partial t}}_{\text{blocked resp.}} + \underbrace{\frac{\partial \Phi_i}{\partial x} \cdot x'(t)}_{\text{motional resp.}} \tag{26}$$

The first term of (26) is due to the so-called blocked (non-motional) impedance of the coil and the second term is caused by the motion, a.k.a. the back-EMF which by the help of (25) can be re-written to:

$$v_{i,emf}(t) = 2Bl_i \cdot x'(t) \tag{27}$$

Note that the forces of the permanent and current dependent cases are quite similar (22) and (25) but differ by a factor 2. Similarly, a factor 2 difference is found for the back EMF: (23) and (27). This factor 2 reflects the difference in storage of energy in the machine: the current dependent components of flux and force store energy during motion. This energy is drawn from the electrical source alongside that which produces the actual force. No such storage of electrical energy happens for the components associated with the permanent field. This factor 2 seems also to have caused confusion in the earlier literature [15] .